Lectures description

- **Taras Banakh** – Basics of algebraic topology
  The mini course will discuss the following topics:
  1) Ordered Geometry: Pasch Axiom, convex sets, polytopes, polytopal and simplicial complexes
  2) Fundamental groups (discrete and continuous, van Kampen, coverings)
  3) Elements of homological algebra (homologies of differential groups, the long exact sequence)
  4) Homology and cohomology groups (Long exact sequence, Mayer-Vietoris, Excision)
  There will be also a lecture "Architecture of Classical Mathematics"
- **Stefan Friedl** – Alexander polynomials and fibered knots
  We will first introduce the linking numbers of knots. We will then introduce Seifert surface and we will use linking numbers and Seifert surfaces to introduce Seifert matrices of knots. These in turn will be used to define the Alexander polynomial of a knot. We will also introduce fibered knots. These are knots such that the complement is a fiber bundle over the circle. We will see that torus knots are fibered. We will also show that Alexander polynomials of fibered knots are monic. This allows us to show that some knots are not fibered.
- **Jarek Kedra** – Braid groups and their applications
  I will define and discuss basic properties of braid groups. For example, I will present an easy proof of the fact that the commutator of a braid group with at least 5 strands is perfect. By joining ends of strands of a braid we obtain a link. We can then use invariants of link and knots to create interesting functions on braid groups. I will discuss applications of this procedure to groups of diffeomorphisms of surfaces.
- **Łukasz Michalak** – From Morse theory to Reeb graphs: topology through critical points
  We will provide an introduction to Morse functions and their role in understanding the topology of smooth manifolds. The relationship between Morse functions, CW-decompositions and handlebody structures will be explored, highlighting how critical points encode the manifold's topology. We will briefly discuss selected classical applications regarding calculation of homology, classification of surfaces, and Heegaard splitting of 3-manifolds. In the second part, we will turn to Reeb graphs, which capture the evolution of connected components of level sets of a smooth function. We will establish their basic properties, especially in the case of surfaces, and also outline connections between Reeb graphs and epimorphisms from the fundamental group of a manifold onto free groups.
- **Sergiy Maksymenko** – Diffeomorphisms of surfaces
  We will discuss the mapping class group of compact surfaces, their generators and algebraic structure for surfaces of small genus.
  As an application we will prove that every compact orientable 3-manifold
  - can be obtained from 3-sphere by "surgery" along some link
  - is a boundary of some simply-connected compact 4-manifold.