ICMU Bayes Intro, Homework 2
Due on September 30th, 2025

1. Recall the set up from the last problem in homework 1. We have $\lambda > 0$ — an unknown rate parameter of a Poisson distribution — and $Y_1, \ldots, Y_n$ are iid samples from this Poisson distribution. We assume the following prior distribution for λ:

$$\lambda \sim \text{Gamma}(s, r); \text{ in this parameterization } \mathrm{E}(\lambda) = \frac{s}{r}.$$

   (a) Using values of s and r you chose while completing homework 1 and data from 10 friends on the number of messages they received during the most recent hour (0, 1, 1, 1, 3, 3, 2, 6, 5, 2),
      i. Compute the posterior mean of λ,
      ii. Compute the 95% Bayesian credible interval for λ.

2. Find the 4th moment ($\mathrm{E}(X^4)$) of a Beta distribution with parameters $\alpha = 2$ and $\beta = 5$ using

   (a) Deterministic integration,

   (b) Monte Carlo integration. Experiment with the number of Monte Carlos samples to get a desired precision.

3. Approximate $\int_0^\infty e^{-x^5} dx$ using

   (a) Deterministic integration,

   (b) Monte Carlo integration.
   Hint: Uniform distribution won't work here, because uniform distribution is not defined on $[0, \infty]$. So I suggest using exponential distribution samples instead. Think about how to write the desired integral in terms of an expectation with respect to an exponentially distributed random variable.