ICMU
INTERNATIONAL CENTRE FOR MATHEMATICS IN UKRAINE

ICMU Winter School
Mathematics of Quantum Matter
5 - 23 January 2026
Kyiv, ICMU Working Space

Abstracts of Courses

Preschool courses:

- Linear algebra: coordinate-free linear algebra, relation to linear algebra in basis, bra-notations. Hermitian operators, unitary operators. Tensor products (with a lot of emphasis on practical computations), partial traces, graphical calculus.

- Quantum mechanics: qubits, Bloch sphere, Hamiltonians, circuits, unitaries, density matrices. Many-body systems examples.

- Abstract algebra: Basics of group theory. Basic representation theory, what group is, what intertwiner operators are. Basic category theory.

- Topology: topology, what is a manifold with boundary, homeomorphism, surfaces, genus of surface, bordisms.

- Basics of complexity theory: Measurement of resources (time, memory, queries, number of gates, etc.), big O notation, complexity classes (P, NP, co-NP, PSPACE, etc.), approximate algorithms and probabilistic complexity (BPP)

Main courses:

Computational aspects of fermionic quantum many-body systems

Barbara Terhal (QuTech & DIAM / TU Delft)

During these lectures we discuss quantum many-body systems, in particular fermionic systems, from a computational perspective. Computational problems pertaining to such systems can be for example determining the ground state energy of the system, or simulating the dynamics of the system and sampling or computing data obtained by measuring observables in the system. Whether such problems are computationally hard or easy can then depend on physical aspects of the system such as the amount of correlation and entanglement. In addition, specific classes of many-body problems can be identified which are computationally easy such as free, ”non-interacting” fermions. The question of computational complexity is also of interest from the perspective of quantum computation: can we more easily solve these problems on a quantum computer?

In these Lectures we will give an introduction to questions of computational complexity for fermionic systems. Topics that we may cover are:

- We review efficient computational techniques used in classical optimization such as linear and semi-definite programming and we review notions of computational complexity.
- We introduce and physically motivate free-fermion Hamiltonians and the complexity of ground-state optimization and simulation. We discuss how fermions, treated in second quantization, relate to qubits via fermion-to-qubit mappings and how one can write down a fermionic quantum circuit or simulate a fermionic Hamiltonian using qubits. What are Slater determinant states and more generally Gaussian states (an example is the BCS state for a superconducting system).
- We discuss fermionic Hamiltonians with interactions. We consider the task of approximate optimization. We review approximate optimization for classical problems such as Max Cut. Can we approximately optimize interacting fermionic Hamiltonians over free fermion states? What do we know about circuits constructed from fermion interactions and how they relate to circuits defined using qubits?

- What is the entanglement entropy for free-fermionic states and what does this measure capture? How does it relate or not relate to entanglement in qubit systems?

In the afternoon tutorials you will be asked to solve some problems using Python and analytical tools.

Some Background Literature

- Python package Free Fermion, https://free-fermion-lib.readthedocs.io/en/latest/installation.html
- Book Convex Optimization by Vanderberghe and Boyd at http://stanford.edu/~boyd/cvxbook/
- Book Quantum Computation and Quantum Information by Nielsen & Chuang (Cambridge University Press).
- J. Surace and L. Tagliacozzo, Fermionic Gaussian states: an introduction to numerical approaches, https://arxiv.org/abs/2111.08343
- F. De Melo, P. Cwiklinski, B.M. Terhal, The Power of Noisy Fermionic Quantum Computation, http://arxiv.org/abs/1208.5334 and references therein.
- M. Wimmer, ACM Trans. Math. Software 38, 30 (2012), https://arxiv.org/abs/1102.3440
- S. Gharibian, Y. Huang and Z. Landau, S.W. Shin, Quantum Hamiltonian Complexity, http://arxiv.org/abs/1401.3916